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On the Impossibility of Reducing the Surplus Approach to a "Special Case" of Neoclassical Theory

A criticism of Hahn in a "Solowian" context

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This article presents some basic characteristics of the Sraffian Surplus approach within a mathematical framework based on the well-known Solow growth model (1956) and widely used among neoclassical economists. A new criticism is then developed within this structure of Hahn's attempt (1982) to prove that the Surplus approach constitutes no more than a "special case" of the Neoclassical model of intertemporal general equilibrium. It is shown in particular that this attempt is vitiated by the fact that Hahn reverses the time axis and thus puts himself in the paradoxical position of determining the past as a function of the future. Various aspects of the Surplus approach cannot be adequately developed within the Solowian framework used and are necessarily ignored here. This drawback is offset, however, by the fact that familiarity and comparative simplicity of the formal structure adopted should help even non-specialists to grasp some distinctive characteristics of the Surplus approach and could perhaps prompt younger economists with an orthodox background to take more interest in it.

JEL code: A2, B0, B4, B51, E11, E12, E13

1. Introduction

Known also as the "Theory of production" or "Neo-Ricardian theory", the "Surplus approach" originated in the work of the Classical economists and Marx and developed more recently around the work of Piero Sraffa (1960) and his successors.[1] Though widely recognized as one of the most solid alternatives to the dominant neoclassical theory,[2] the approach seems to have run up against problems of communication over the last few years, especially as regards the younger generations of economists. While there is no lack of clear and authoritative introductions to this specific area of economic theory,[3] it has been ironically noted that these seem to reassure "believers" but almost never get through to "unbelievers".[4] In other words, economists with a neoclassical background remain largely indifferent and show little interest in meaningful discussion with some of their most formidable critics. It is no easy matter to ascertain the causes of this disregard for the Surplus approach, and this is hardly the place to address delicate questions regarding the sociology of knowledge and power. It should, however, be borne in mind that the neoclassical vocabulary is now more predominant in academic circles than ever before and that its hegemony tends to be consolidated cumulatively through control over the training of new generations. This trend appears to be so strong today that there is even talk of neoclassical "imperialism".[5] Given this situation, it is worth considering whether advocates of the Surplus approach could benefit in the initial phases of discussion with scholars of the orthodox persuasion from the use of terminology closer to the mainstream at the specific level of exposition. It may well be in fact that a more orthodox vocabulary could serve to convey some basic characteristics of the alternative theory with greater immediacy and thus arouse more interest among economists with a neoclassical background, especially the young. The use of this terminology could therefore be seen in a certain sense as a sort of Trojan horse in debate on contemporary economic theory, making it possible for theorists of the Surplus approach to infiltrate the neoclassical citadel by speaking the same language as the inhabitants and perhaps even to make a few more "converts" amongst them.

Our aim here is to put forward one possible application of this particular communicative strategy. We shall start by showing that the Sraffian Surplus approach can be presented in its essential terms within a mathematical framework drawn from the well-known Solow growth model (1956) and widely used among neoclassical economists. A new criticism will then be developed within this structure of Hahn's attempt (1982) to prove that the Surplus theory constitutes no more than a "special case" of the neoclassical model of intertemporal general equilibrium. In particular, as we shall see, Hahn's attempt is vitiated by the fact that he reverses the time axis and thus puts himself in the paradoxical position of determining the past as a function of the future. Many aspects of the comparison of the two theories cannot of course be adequately developed within the Solowian framework adopted here and will necessarily be passed over. For one thing, the mathematical system adopted will describe an economy that produces only one good and thus precludes any examination of Sraffa's important criticism of the

neoclassical theory of capital. For another, it envisages a continuous and differentiable production function with perfect substitutability of the factors of production, which has always been considered wholly unrealistic by Sraffian theorists. Moreover, it assumes constant returns to scale, which Sraffa specifically indicated as unnecessary for the purposes of his analysis. Advocates of the Surplus approach who attach more importance to safeguarding the integrity of the theory than facilitating communication of its essence will thus have legitimate grounds for deciding that this is too high a price to pay. At the same time, however, the familiarity of the formal structure adopted here should help non-specialists to understand the basic reasons why the Surplus approach is logically alternative to neoclassical theory and could perhaps arouse some interest among younger economists with an orthodox background.[6]

2. The Surplus approach in Solowian dress

Let us start by ascertaining whether the basics of the Sraffian Surplus approach can actually be presented within a formal structure drawn from Solow (1956). In particular, we shall now address the question of whether the key ideas of Surplus theorists can be also described within the framework of a model of development with a single good and flexible coefficients based on a Solowian mathematical structure. To this end, we shall present the stylised description of a capitalistic system with no foreign trade producing a single good by means of labour and the good itself. Let us begin by describing the technology of the system. Let K be the physical amount of the good available as capital and therefore used as production input, L the quantity of homogeneous labour employed and X the quantity of the good produced. We thus obtain the following production function:

$$X = F(K, L)$$

On the assumption that the production function has constant returns to scale, it is possible to state that:

$$\alpha F(K, L) = F(\alpha K, \alpha L) \qquad \forall \alpha \in R^{+}$$

By positing $\alpha = 1/L$, the function can be rewritten as follows:

$$x = f(k)$$

where $x = f(k) = X/L$ represents output per unit of labour and $k = K/L$ capital per unit of labour, i.e. the technique of production adopted. It should be noted that once k has been determined, the ratio between K and X is also known because $K/X = k/f(k)$.

Let W be the monetary wage, r the rate of profit, P the level of monetary prices of the good produced and $Y = PX$ the monetary value of the output and hence income. The value of the output achieved will be equal to:

$$Y = PX = WL + (1+r)PK$$

This expression can be used in various ways. In our case, division by P and L gives us the distribution of the physical output between wages and total gross profits prior to replenishment of capital for every given unit of labour:

$$(1) \qquad f(k) = \frac{W}{P} + (1+r)k$$

Let us now go on to examine the coefficient k, which can be regarded as fixed or flexible depending on the assumptions as regards technology. With a view to facilitating comparison with the neoclassical analysis, it is assumed here that the production function is continuous and differentiable and that it meets the following conditions:

$$f(0) = 0, \;\; f'(k) > 0, \;\; f''(k) < 0$$

As k can therefore assume infinite values, a problem arises as regards choice of the optimal technique. This can be determined once the rate of profit is given. Firms will in fact tend to opt for the technique k that maximises the difference:

$$\max \quad f(k) - (1+r)k$$

which indicates output net of the amount of the rate of profit, all in terms of units of labour. Maximisation means that:

$$(2) \qquad f'(k) = 1 + r$$

In other words, firms will opt for the particular technical combination k of capital and labour ensuring fulfilment of the maximum condition described by equation (2).

Equations (1) and (2) are formally identical to those contained in the Solowian growth models and will therefore be familiar to neoclassical economists. They will also appear very far removed from the typical characteristics of the Surplus approach at first sight. Suffice it to consider the assumption that the production function is continuous and hence that k can assume infinites values. This hypothesis forms no part of the theoretical tradition of the Surplus approach and has indeed been criticized quite frequently by Sraffian economists. It should, however, be pointed out that these distinguishing elements are not crucial in the sense that the general relations between the exogenous and endogenous variables are more important for our understanding of the structure of

a theory than the specific functional forms adopted.[7] On given assumptions, equations (1) and (2) can therefore provide a schematic representation of some basic characteristics of the Surplus approach. The system to be addressed thus consists of two equations with four unknowns: r, k, W and P. On the assumption that the rate of profit r is exogenous, equation (2) makes it possible to determine the optimal technology k that firms will choose. With r given and k known, equation (1) will make it possible to determine the real wage W/P. Finally, for every given monetary wage W, it will also be possible to obtain the corresponding level of monetary prices P. It should be noted that the entire sequence makes sense on the assumption that once the technology k has been determined, the ratio between the quantities K and L can adapt to it.

Sraffian theory is extremely sophisticated and there can be no question of encapsulating its essence in this pair of simple equations. Some important characteristics of the Surplus approach can, however, be pinpointed in the particular sequence described above. First, it is assumed that a distributive variable is exogenous. This means that it can be determined by a complex of social, political and institutional forces that the various authors considered regard as capable of ranging from conditions of macroeconomic equilibrium and mechanisms of monetary policy to the more general state of the balance of power between the social classes. Whatever these factors may be, they are in any case all crystallized initially within the exogenous distributive variable and excluded from the analysis. The optimal technique of production can thus be determined in relation to this variable, once it has been set, and the other distributive variable can then be determined residually. Readers with a neoclassical background are invited to note that this sequence is based on the idea that it will always be possible for production inputs to adapt to any given distribution of income once the corresponding technique of production is known. In accounting for this capacity to adapt, it should be borne in mind that the Surplus approach is not based on given endowments of production inputs and, unlike the neoclassical theory, does not assign the distributive variables the task of ensuring that these endowments are wholly absorbed by their respective demand. The Surplus theorists instead maintain that the means of production can vary as regards degree of utilisation and that in any case they are themselves products. Moreover, they also usually regard labour as abundant most of the time with respect to the productive requirements of the system. All this means that the Surplus approach generally rules out any possibility of production inputs proving scarce and hence capable of influencing the determination of distribution and the other variables. While situations can nevertheless exist in which constraints of a quantitative nature are generated within the system, any connection with prices and distribution would, however, be completely different from that envisaged by neoclassical theory.

The key differences between the Surplus approach and neoclassical theory are therefore immediately obvious. The former establishes a distributive variable exogenously with no reference to the technique of production and endowment of production inputs. On the contrary, as we shall see, the latter always takes the initial endowments of production inputs as its starting point and uses the relations

between these endowments and their respective demand to determine prices and all of the distributive variables endogenously. In short, while advocates of the Surplus approach determine the distribution of income on the basis of forces external to the analysis and even to class conflict, neoclassical economists regard the scarcity of factors of production in relation to the demand for the same as the crucial element upon which the calculation of prices and distribution must always be based.

3. A new criticism of Hahn

Two different views of scarcity and prices thus seem to establish the existence of an unbridgeable gap between neoclassical theory and the Surplus approach. At the same time, however, not everybody would agree that the two theories are logically incompatible. In this connection, and remaining within the sphere of the Solowian formal structure, we can profitably focus attention on Frank Hahn's well-known article *The Neo-Ricardians* (1982). Regarded by many as one of the most influential contributions to 20th-century debate on economic theory,[8] it constitutes the most authoritative attempt put a definitive end to the well-known dispute between the two schools of thought at Cambridge on the theory of capital,[9] aiming explicitly at securing a clear victory for advocates of the neoclassical approach. Hahn maintains in his article that Sraffa's criticism of the theory of capital fails to identify any logical flaw in the "short-period" version of the neoclassical analysis based on the model of intertemporal general equilibrium. Furthermore, Hahn endeavours to prove that the Surplus approach inspired by Sraffa's views constitutes no more than a largely insignificant "special case" of the neoclassical model of intertemporal equilibrium.[10]

Hahn's approach and conclusions still appear to enjoy widespread acceptance in academic circles[11] and were explicitly reasserted by the author himself quite recently.[12] It has, however, been demonstrated that his analysis is vitiated by a series of serious errors of logic and method. It has been argued that the goal of reducing the Surplus approach to a "special case" of the neoclassical model is flawed from the very outset because these are theoretical constructions based on two sets of hypotheses and exogenous variables that are completely different and mutually incompatible. Among other things, it is also suggested that the short-period versions of neoclassical analysis are also vulnerable to Sraffian criticism of the theory of the capital, contrary to Hahn's claims, and indeed that the neoclassical theory of intertemporal equilibrium could instead be considered a "special case" of the Sraffian theory of the prices.[13]

Though rigorously argued and in many respects unanswerable, these criticisms appear to have been largely ignored in mainstream economics. Once again, there seems to be a problem of non-communication. This is why we shall seek here to remain within the framework of the well-known Solowian mathematical structure so as to present a further criticism of Hahn's argument in

familiar and accessible language. We shall argue that in attempting to reduce the Surplus approach to a "special case" of the neoclassical model, Hahn makes a number of serious theoretical errors that lead him to reverse the time axis so that the past is paradoxically determined as a function of the future.[14]

4. The neoclassical "general case"

Let us examine how Hahn (1982) endeavours to interpret Sraffian Surplus theory as no more than a "special case" of the "general" neoclassical analysis. As we shall see, Hahn rejects a priori the idea that a distributive variable can be regarded as exogenous and above all that it can be considered independent of the endowments of productive factors.

Faced with equations (1) and (2) in their Sraffian interpretation, Hahn would maintain that they in no way represent the basis of an alternative system but correspond instead to the equations of a typically neoclassical model whose only peculiarity lies in the absence of the equation needed to determine *r* endogenously. He therefore sets out in search of the "missing equation" (Hahn 1982, section 5) with a view to constructing a neoclassical model defined as "general", i.e. capable of encompassing what he regards as the Sraffian "special case" and hence of determining the rate of profit and all the other distributive variables endogenously. To this end, he constructs a system of intertemporal general equilibrium representing an economy that has only two periods. As presented in the original article, the system produces two goods. As stated above, however, our aim in this article is to make the analysis more accessible by remaining within the framework of the widely known Solowian formal structure (1956). This is why Hahn's arguments are faithfully reported below but adapted to the context of a Solowian model with only one good. In this particular sphere, the equation corresponding to the "equilibrium condition for producers" contained in Hahn's "general" neoclassical model can be rewritten as follows:[15]

$$P_t^{'} X_t = W_t^{'} L_t + P_{t-1}^{'} K_t$$

It should be noted that the prices *P'* are not expressed in money here but in terms of the good at time *t*:

$$P_t^{'} = \frac{P_t}{P_t}, \qquad P_{t-1}^{'} = \frac{P_{t-1}(1+i)}{P_t}, \qquad W_t^{'} = \frac{W_t}{P_t}$$

Hahn's equilibrium condition initially appears to bear no resemblance to the equation (1) adopted here. However, if all of his relative prices are multiplied by the monetary price P_t, the condition becomes:

$$P_t X_t = W_t L_t + (1+i) P_{t-1} K_t$$

Furthermore, given the typical arbitrage condition (Solow 1956):

$$1 + r = \frac{P_{t-1}(1+i)}{P_t}$$

we have:

$$P_t X_t = W_t L_t + (1+r) P_t K_t$$

and equation (1) is easily obtained by dividing the whole by $P_t L_t$. We have thus shown that the condition of equilibrium contained in Hahn's (1982) "general" model corresponds exactly to equation (1). Moreover, the assumption that it is possible to choose from a variety of technical combinations *k* allows us to add equation (2) to (1).

The "general" neoclassical model is, however, not yet complete. It is in fact still necessary to find what Hahn describes as the "missing" equation in order to close the system. To this end, the equation of equilibrium between production and demand for goods can be introduced into the analysis. Let *C* and *I* be the aggregate monetary values of consumption and investment respectively. We can now write:

$$Y = C + I$$

Let us also define aggregate saving as $S = Y - C = sY$, where *s* is the average propensity to save of the population. Remembering that $Y = PX$, assuming that $I = (1+g)PK$ and dividing the whole by *PL*, we can now rewrite the equation as follows:

$$(3) \quad sf(k) = (1+g)k$$

The model (1), (2) and (3) can be understood as the single-good version of Hahn's neoclassical "general case". The system is made up of three equations with five unknowns - namely *r*, *W*, *P*, g and *k* - with s representing a parameter determined by the habits of the population and consequently assumed as given. The solution is as follows. As the endowments of the inputs of capital and labour *K* and *L* available at the beginning of every period are exogenous, their ratio *k* also proves to be determined. The system is therefore complete with (2) determining *r*, (1) determining *W/P* and (3) determining *g*.[16] Finally, on the typically neoclassical assumption that *P* is given by quantitative theory, the monetary wage *W* of equilibrium can also be obtained from (1).

Let us now look beyond the mathematical solution characterising this "general" model and attempt to describe its internal mechanism. The model describes an economic system in which, at the beginning of each period, while

families offer the endowments of labour L and capital K accumulated over the previous period on the factor markets, firms express demand for these endowments, carry out the production process and make the output available. The same families then express demand for the production of the firms and divide it between consumption and saving. It should be borne in mind that the model regards saving as wholly transformed into investment, i.e. the accumulation of capital. This means that equation (3) is to be read in typically neoclassical terms from left to right. The entire mechanism can be described by means of an elementary Walrasian equilibrium of production and exchange. Within it, the behaviour of families is extremely rigid. They supply all the factors at their disposal, demand the entire product and divide it in fixed proportions between consumption and accumulation on the basis of the propensity to save s, which is assumed as given than in this model. Among other things, this makes it possible to examine the situation of families as a uniperiodal equilibrium, since there is no optimal behaviour as regards accumulation, i.e. the choices connecting the present with the future. Optimisation does exist, however, and is incorporated into the demand for factors on the part of firms. This demand derives from the programme to maximise extra profits. Let us assume that extra profits are zero in a state of equilibrium (otherwise constant returns would lead to null or infinite demand for factors and supply of products). Given the market prices r, W and P, the programme determines the optimal demand for L and K on the part of each firm in correspondence with the equality between the partial derivatives of the production function with respect to the various factors and their prices. The optimum condition contained in equation (2) therefore derives from this programme. The point is, however, that the endowments of L and K made available by the families and therefore also their ratio $k = K/L$ are all exogenous data at the market level. There is thus nothing to guarantee that these data will coincide with the firms' demand for factors. As long as there is a discrepancy between given demand and supply, however, the prices W/P and r will tend to change. The price of the comparatively scarce factor will tend to increase and vice versa. These changes will act on the levels and composition of the firms' demand for factors until they come to coincide perfectly with the given supply. It will therefore be understood that in this analysis the distributive variables assume the role of indicators of the scarcity of factors with respect to demand. As regards the specific mechanism of equilibrium, it should be borne in mind that the production function is assumed to present constant returns to scale. This means that if the firms' extra profits are positive, the demand for L and K and the supply of X will tend toward infinity, thus causing increases in W/P and r. The opposite will happen if the extra profits are negative. They will ultimately cease to exist precisely when equilibrium is attained between the absolute levels (and hence also the proportions) of demand and supply for endowments.[17] Among other things, this mechanism helps us to understand how prices move until the point of complete product exhaustion between wages and profits, and therefore clarifies the determinants of equation (1), upon which this exhaustion is based.[18] The attainment of equilibrium on the factor markets obviously leads also to determination of the level of production, which will be divided by families between immediate consumption and the

accumulation of capital for the following period. It should be noted in this connection that the rate of capital accumulation will only coincide with the rate of growth of the working population by chance, which means that the ratio k of capital to labour will tend to change in the following period.

The "general" model described here thus takes the endowments of factors given at the beginning of the period as its starting point and determines an equilibrium in which the physical magnitudes do not necessarily grow at the same rate. For these reasons, the model can be made to correspond to an equilibrium described as "non-stationary" within the framework of Solow's analysis and usually termed "short-period" in the context of the intertemporal models.[19]

5. The Sraffian "special case"

Hahn regards the model described above as showing that the neoclassical theory is capable not only of encompassing equations (1) and (2) but also of determining the rate of profit r, which Sraffian economists instead prefer to regard as exogenous. This result cannot, however, be considered satisfactory as yet for Hahn's purposes. If the neoclassical model is to have the character of a "general case", he must in fact show that it can determine r in compliance with a key assumption of the Sraffian Surplus approach, according to which the distribution of income can be determined independently of any reference whatsoever to the technique of production and production inputs used. This is not in fact true of the "general" model as described so far, since r turns out to be a function of k and hence ultimately dependent on the technique of production and the initial endowments of K and L. Hahn accounts for this, however, on the grounds that the Surplus approach is only a "special case" and that conclusions compatible can be obtained simply by imposing suitable constraints on the "general" neoclassical model (Hahn 1982, section 5). To this end, he first suggests the introduction of a particular assumption as regards accumulation decisions. Usually referred to as the "classical saving hypothesis",[20] this is in fact widely used by theorists of the Surplus approach. In its extreme version, the hypothesis rests on the idea that the society is divided into two social classes, namely capitalists and workers, and that while the former save all of their profits, the latter consume all of their wages. This means that aggregate saving corresponds to total profits:

$$S = (1+r)PK$$

Secondly, Hahn's solution involves recourse to equation (3) of macroeconomic equilibrium also for the "special case".[21] Given the classical saving hypothesis and the customary division by P and L, the macroeconomic equilibrium becomes:

$$(1+r)k = (1+g)k$$

from which it is possible through simplification to obtain:

(3') $r = g$

The model of the Sraffian "special case" is therefore made up of equations (1), (2) and (3') and the five unknowns *r*, *W*, *P*, *g* and *k*. At this point, if it is assumed this time that *g* is exogenous and *k* endogenous, a new resolutive sequence is obtained. Equation (3') determines the rate of profit *r* exclusively on the basis on the growth rate *g*. Therefore, once *r* is known, the technical combination *k* is determined by (2). And it should be made clear that once *k* has been determined, the factorial endowments *K* and *L* will have to prove compatible with it. Finally, with *P* given once again by quantitative theory, it is also possible to obtain *W* from (1).

Hahn therefore believes he has proved that it is possible, by applying suitable constraints, to determine a "special case" from the "general" neoclassical model capable, in his view, of satisfying all the conditions of the Sraffian Surplus approach, and especially the prerequisite of an *r* independent of the production technique and initial endowments of production inputs.[22] He insists, however, on adding that this "special case" is insignificant because the result at which it arrives can only be regarded as consistent purely by chance, i.e. only when the ratio of the initial endowments of *K* and *L* happen to coincide with the term *k* determined as a function of the rate of profit and therefore ultimately of the exogenous rate of growth. These elements can, however, coincide only by chance, since there is no logical reason why they should necessarily do so. This is why Hahn ironically concludes that if this coincidence does not come about, it means that Sraffa is "out of luck" together with all the theorists of the Surplus approach.[23]

6. The paradox of the past as a function of the future

We have already mentioned the logical errors identified by various scholars in Hahn's argument. Our purpose here is to raise the further objection that the so-called "special case" does not comply with the Sraffian canons because the rate of profit is not determined independently of the technique of production and endowments of production inputs. This will become clear on examination of the solution envisaged by this "special case", which takes the assumption that the rate *g* of capital accumulation is exogenous as its starting point. In the neoclassical sphere, however, accumulation depends on the savings available. Therefore, if *g* is exogenously determined, a certain amount of savings is implicitly required in order to generate that particular rate of accumulation. If a classical saving hypothesis is then assumed, as it is in Hahn's argument, attainment of the required amount of saving will in turn necessitate a certain rate of profit. This means that *g*

determines saving unequivocally and therefore determines *r* on the basis of equation (3'). The point is that, as derived from equation (2), *r* in turn determines the optimal ratio *k* at which firms will employ the factors of capital and labour. If *k* is already determined, however, the problem arises of its compatibility with the endowments of *K* and *L*. It now becomes clear that this compatibility can only be attained if at least one factor endowment adapts passively to the other variables. This is obviously absurd, however, since factor endowments should all be treated in the neoclassical equilibrium as exogenous data inherited from the past. In short, the entire procedure moves somehow backward. Hahn is so set on encompassing the Sraffian analysis within neoclassical theory that he gets himself into the paradoxical position of reversing the temporal axis so that the past is determined as a function of the future. It should be noted, however, that this does not mean that the rate of profit is independent of the technique of production and endowments. As Hahn himself admits, it indicates that what he describes the Sraffian "special case" will only make sense in a wholly fortuitous set of circumstances where the past "history" of the system turns out by chance to be compatible with an exogenously determined rate of accumulation.[24]

In short, Hahn's neoclassical reinterpretation of the Surplus approach produces a grotesque result. He refuses in fact to admit that the Sraffian approach determines prices and distribution in terms of a logic in which the scarcity of factors plays no role whatsoever and that it is therefore incompatible with the neoclassical formulation. His stubborn insistence on attempting to enclose the Surplus approach within an orthodox model leads him to accuse the Sraffian economists of devoting their energies to incomprehensible exercises based on reversal of the time axis, an interpretation that is bizarre to say the least. As we have seen, however, the reality is that the theorists of the Surplus approach do not regard the distributive variables as depending on the endowments of production factors and do not assign them the function of balancing demand with these exogenous endowments. Their idea is rather that the distribution of income can be determined prior to and independently of production inputs, since these are not given but can in fact adapt to it. Finally, it is worth pointing out that the paradoxical character of Hahn's demonstration shows that it is impossible for neoclassical theory to incorporate not only the Sraffian conception of distribution but also the Keynesian analysis of the level of income. In actual fact, Hahn's "special case" proves not only incompatible with the hypothesis that distribution is independent of production endowments and technique but also wholly misleading with respect to the interpretation of any Keynesian link between investments and savings. From a Keynesian viewpoint, the exogenous character of the rate of accumulation *g* indicates that demand determines income and saving as well as the rat of profit, given the classical saving hypothesis. When Keynes and the Keynesians maintain that investment determines saving, they are therefore saying that demand generates the ensuing income. Hahn's "special case" describes a neoclassical model, however, where the assumption that the rate of accumulation is exogenous leads to a completely different and paradoxical situation in which demand "constrains" previous saving.[25]

7. A switch to stationary growth changes nothing

Hahn's "special case" refers to a "non-stationary" or "short-period" equilibrium. The Surplus approach instead regards a "long-period" position in that the production inputs are assumed to be capable of adapting to the technique chosen in relation of the rate of profit. It is therefore clear that in addition to the various theoretical incongruities pointed out by critics, Hahn can also be charged with the methodological error of interpreting a long-period analysis in a short-period context. It should be pointed out, however, that the shortcomings of Hahn's demonstration would not be rectified by switching the point of reference to a neoclassical equilibrium over a longer period, such as the Solowian equilibrium of stationary growth. In order to prove this, let us examine a Solowian model that incorporates the classical saving hypothesis and is therefore again based on the equations (1), (2) and (3'). The equilibrium of stationary growth of this model envisages no change in the ratio of capital to labour, which means that the rate g of capital accumulation must be equal to the rate n of growth of the working population, which is assumed to be exogenous. Given that g is thus equal to the exogenous rate n, r is obtained from (3') and k from (2). Once k is known, this type of equilibrium obviously entails adaptation of the ratio of L to K. The similarity to the solution admitted by the Surplus approach is evident at the formal level. In this case, however, the passive adaptation of the ratio between capital and labour does not constitute a violation of the key characteristic of neoclassical theory, whereby prices and distribution must always be determined by the scarcity of production factors with respect to demand. Here it simply indicates that the state of stationary growth is attained after a series of non-stationary short-period equilibriums. In order to prove that the scarcity of factors constitutes the basis for the determination of prices and distribution also in this case, we shall examine the way in which Solow's model with the classical saving hypothesis converges from a non-stationary to a stationary equilibrium. At the beginning of every period, as is customary, the existing endowments of K and L must be regarded as given, and therefore k is also given. This means that the prices r and W/P determined on the factor market will be such that the firms' demand corresponds to exactly to those factor endowments. Once r is known, the classical hypothesis will make it possible to determine saving immediately and with it also the rate g of capital accumulation. In other words, in non-stationary conditions equation (3') must be read from left to right, i.e. assuming that g is the endogenous variable. Except by chance, however, g will differ from n, the exogenous rate of growth of the workforce. If we assume for example that $g < n$, the endowment of labour grows more than the capital endowment, and the factor inputs K and L will therefore be available in a lower ratio k in the following period. As a result, capital will be comparatively scarce and its marginal productivity comparatively. This means, on the basis of equation (2), that the equilibrium rate of profit r will be higher, which will lead in turn to an increase in saving and hence also in the rate g of capital

growth. Under the hypotheses on technology stated in section 2, the process is convergent and will continue until the attainment of a given $g = n$. Once the equilibrium of stationary growth has been attained, the prices will be such as to guarantee both equilibrium between factor supply and demand and equality between their respective rates of growth. It will be clear at this point that the endogenous character of the ratio between K and L represents the formal outcome of a mechanism that continues to be based on the customary neoclassical principle of the scarcity of factors and certainly does not constitute a violation of the same.

8. A graphic comparison

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[1] Some economists have used the term "Neo-Ricardian" in a negative sense to call the connections between this approach and the work of Marx into question. See for example Rowthorn (1974). Among the authors maintaining the Marxian ancestry of the approach, see Garegnani (1984). For the link between Marx and the Sraffian approach, see also Steedman (1977). For a reconstruction, see Kurz and Salvadori (2008), Aspramourgos (2004).
[2] There are moreover various reasons to maintain that there are far more affinities than divergences between the Surplus approach and the other major streams of critical thinking. See Brancaccio (2005), for example, for an analytical synthesis of the Surplus approach and the monetary circuit theory.
[3] See, for example, Kurz and Salvadori (1995) and Pasinetti (1975).
[4] Bliss (1996).
[5] See Pagano (2004) for a recent example.
[6] This hope is actually based on a concrete experience in 1999, during a series of seminars for young economists at the LSE and UCL in London, when I used a formal structure drawn from Solow (1956) to introduce some mainstream colleagues to the debate between neoclassical and Sraffian economists. It seems reasonable to believe that the success of this initiative stemmed precisely from the use of the only mathematical framework immediately recognizable and acceptable to the young doctorate students involved. This is not to say that this communicative expedient is sufficient in itself. On the contrary, there is reason to believe that it may have prompted some of those young economists to study the classical works of the Surplus approach in depth. It should in any case be noted that there is more than one precedent for the experiment carried out here. The mathematical structure drawn from Solow (1956) has in fact already been used as a testing ground for alternative theoretical systems. See Hahn and Matthews (1964) and Darity (1981), among others. The new ground broken in this article regards the fact that the simple Solowian formal structure is used to discuss a controversial point hitherto addressed solely within the framework of multisectoral models and models of intertemporal general equilibrium.
[7] Dobb (1973).
[8] For the influence of Hahn's article in the theoretical debate, see Harcourt (1990).
[9] See Pasinetti (2000) and Petri (2004).
[10] Some of these concepts had already been expressed by Hahn (1975). See Bliss (1975) for a similar approach.
[11] See Mandler (1999) for a recent example.
[12] Hahn (2003).
[13] Per queste ed altre critiche, si vedano tra gli altri Dumenil e Levy (1985), Schefold (1985), Garegnani (1990), Kurz e Salvadori (1995), Pasinetti (2000), Garegnani (2003), Petri (2003).
[14] Garegnani implicitly identifies this error when he states that Hahn uses the neoclassical short-period analysis in a peculiar way, which consists "of regarding physical compositions of the capital endowment the economy moves away from, as equivalent [...] with physical compositions

the economy tends to" (Garegnani 2003, p. 150). Garegnani does not, however, consider in depth the final phases of the operation carried out by Hahn, which consist, as we shall see, in linking the rate of profit to an exogenous rate of growth..

[15] The point of reference is Hahn's equation (3.18) (1982, section 5).

[16] This theoretical structure coincides at the conceptual level with the one represented by Hahn's equations (3.17), (3.22), (3.25). See Hahn (1982, section 5).

[17] This process of adjustment can also be described by transforming the problem of the non-constrained maximisation of extra-profits into a problem of minimisation of costs for the production of a given quantity X_0. The problem can be expressed as follows: $min\ (W/P)L + (1+r)K$ $sub\ F(K, L) =$ given X_0. The solution reveals that $(W/P)/(1+r) = dX/dL/dX/dK$. We know that the ratio $(W/P)/(1+r)$ corresponds to the slope of the path of expansion. If we consider the quantities of labour and capital given by L_0 and K_0, it is possible, for example, that the slope of the path will give rise to the full utilisation of labour but not capital. This will lead to a change in prices altering the slope until $(w/P)/(1+r) = L_0/K_0$, in other words, until both factors are fully utilised. See Mas Colell et alii (1995, chapter 5).

[18] See Burmeister and Dobell (1970) for a formal representation of the theorem of product exhaustion.

[19] Some clarification is called for as regards the connection between "non-stationary" and "short-period" equilibrium, which involves a logical link between Solow's analysis and the modern analyses of intertemporal and temporary equilibrium. It should be recalled Solow subjects the behaviour of agents to marked degree of simplification so as to eliminate the problems regarding optimal intertemporal allocation and expectations. The temporal structure of the model is, however, analogous to its more complex counterpart in the modern analyses of neoclassical general equilibrium, and it is for this reason that we have been able to incorporate the operation carried out by Hahn (1982) within it. At the same time, it should be specified that the analogy between "short-period" equilibrium and non-stationary Solowian equilibrium is permissible because the model presented here has only one good and therefore treats capital as a homogeneous physical magnitude. If capital were instead expressed in terms of value, a point of non-stationary equilibrium with K given and a unique and uniform rate of profit would have to be regarded as a long-period equilibrium in the classical and traditional neoclassical sense of the term (Garegnani 1979). The equilibrium of stationary growth of the Solowian model would instead correspond in that case to an equilibrium described by classical and traditional neoclassical economists as "secular". See Garegnani (1976) and Petri (1999). It should of course be borne in mind that with K expressed in terms of value, Solow's model would be subject to the Sraffian criticisms of the neoclassical theory of capital and hence prove wholly inconsistent.

[20] Hahn and Matthews (1964).

[21] It would in fact appear somewhat imprecise to describe a system to be closed with an equation of equilibrium between income and aggregate expenditure as "Sraffian". In formal terms, it is unquestionably possible to combine a Sraffian system of production prices with an equation of macroeconomic equilibrium. In particular, given the classical saving hypothesis and the assumption that the rate of growth is exogenous, it is possible to obtain the rate of profit from the macroeconomic equilibrium. Given the latter, it is then possible to arrive through the system of production prices at the determination of wages and the relations of exchange between goods. This solution has, however, been put forward only by some of Sraffa's successors, such as Pasinetti and Salvadori, and Pasinetti himself points out that it constitutes only one of the various possible formal closures of a Sraffian system (Pasinetti 1990). Others have instead subjected this procedure to marked criticism based on the idea that Sraffa's exogenous distributive variable – which can be the rate of profit or wages – refers to a "normal" distribution in the classical sense, which is assumed to be characterised by a certain degree of "persistence" and cannot therefore be regarded as directly dependent on the constant change of macroeconomic variables. Critics have also pointed out that this procedure unduly restricts dynamic analysis of the quantities produced, confining it exclusively to the case of stationary growth. They have therefore indicated alternative

ways of restoring macroeconomic equilibrium that are based no longer on variations in the rate of profit but rather on change in the degree of utilisation of productive capacity or the amount of autonomous expenditure that does not generate additional capacity. See Brancaccio (2003) for an overview and Brancaccio (2005) for an analytical synthesis of the two positions. In any case, Hahn does not take this internal Sraffian debate into account, but frequently appears to believe that the connection between the macroeconomic equation, rate of profit and production prices can be associated with "Sraffians" or "Neo-Ricardians" in the broad sense.

[22] The procedure that Hahn (1982) encloses in the system of equations from (3.22') to (3.30) is thus presented within the framework of a model with a single good.

[23] In actual fact, Hahn (1982, section 5) uses this expression with reference to the problem of the uniformity of rates of profit rather than the compatibility between the rate of accumulation, rate of profit and optimal ratio of physical capital to labour. The terms of the problem are, however, completely equivalent at the conceptual level in the sense that Hahn's expression refers in any case to what it he regards as the primary limitation of the Sraffian Surplus analysis, namely an unavoidable – except by chance – incompatibility between the endowments and the other exogenous variables of the model.

[24] Hahn (1982, section 5).

[25] Hahn seems to realise the problem when he states that «there is an interpretation of g connected with the question of "animal spirits" that could cause the neoclassical theory some difficulties». The question is then hastily dismissed, however, with a remark to the effect that «problems of interpretation have no importance in the case considered» (Hahn 1982, section 5). This is, however, somewhat perplexing, as Hahn's article is wholly concerned with putting forward a neoclassical interpretation of the Surplus approach.